TMUA Mock Paper 2

20 Questions in the style of a TMUA Paper 2

75 Minutes

No calculator allowed

Enjoy!

(By yotta)

Q1. Which of these statements is true?

(A) $\sqrt{361}$ is irrational

(B) $\sqrt{x^2} = x$ for all real x

(C) $f(x) = 3x^2 - x + 5x^{\frac{1}{2}} + 1$ is a polynomial function

(D) $\frac{195}{13}$ is an integer

(E) $2 + 2 = 5$

(F) $\ln(-1) = \pi$

Q2. Which of these statements is true for positive integers n:

1. n is prime **if** $n = 6k + 1$ **or** $n = 6k - 1$ for some integer k

2. n is prime **only if** $n = 6k + 1$ **or** $n = 6k - 1$ for some integer k

(A) Neither

(B) 1 only

(C) 2 only

(D) Both 1 and 2

Q3. Which of these statements is true about $x = 21222324252627$:

1. x can be written as $p^2 + q^2$ where p and q are 2 distinct positive integers

2. x can be written as $p^2 - q^2$ where p and q are 2 distinct positive integers

3. x can be written as p^2q^6 where p and q are 2 distinct positive integers

(A) None

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

Q4. $f(x)$ is a function defined for **all real** x.
Here are 3 statements:
J: $f(3) = 1$, and $f(5) = -2$
K: $f(x) = 0$ has exactly 3 solutions in the interval $3 < x < 5$
L: $f(x) = 0$ has an odd number of solutions in the interval $3 < x < 5$
Here are 3 more statements:
R: K is necessary for J
S: K is sufficient for J
T: L is necessary for J
Which of statements R, S and T are true?

(A) None

(B) R only

(C) S only

(D) T only

(E) R and S only

(F) R and T only

(G) S and T only

(H) R, S and T

Q5. Find the sum of the x-coordinates of the points of intersections of

$$y = (\sqrt{x} - 3)(\sqrt{x} + 3)$$

and

$$|x| = \frac{y + 20}{3}$$

(A) There are no points of intersection

(B) -11.75

(C) -3.5

(D) -2.75

(E) 0

(F) 2.75

(G) 5.5

(H) 15.25

Q6. $f(x)$ is a polynomial function defined for all real x.
Statement P: $f'(5) = 0$
Statement Q: There is a turning point at $x = 5$
Which option is true?

(A) P is neither necessary nor sufficient for Q

(B) P is necessary but not sufficient for Q

(C) P is sufficient but not necessary for Q

(D) P is necessary and sufficient for Q

Q7. A, B and C are three points on a regular n-sided polygon, where $n \geq 3$. Let O be the centre of the circle that has A, B and C on its circumference. In radians, $\angle AOB = \frac{5}{6}\pi$ and $\angle BOC = \frac{2}{5}\pi$. Then n is necessarily a multiple of k. What is the largest value of k such that this statement is true?

(A) 240

(B) 120

(C) 60

(D) 30

(E) 12

(F) 6

(G) 5

Q8. Find a counterexample, if it exists, to the statement:
$\int_0^1 f(x)\,\mathrm{d}x$ is equal to the area enclosed by $f(x)$, the x-axis, $x = 0$ and $x = 1$ **if** $f(x)$ is defined for $0 \leq x \leq 1$

(A) $f(x) = (x-2)^2$

(B) $f(x) = (\sin(x)+1)(\sin(x)-1)$

(C) $f(x) = \ln(x)$

(D) $f(x) = 1 - x$

(E) $f(x) = \cos(x)$

(F) The statement is incorrect, but none of the above are counterexamples

(G) The statement is correct, so none of the above are counterexamples

Q9. It is given that $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$. Here is an attempt to prove $0 = 1$:

1. Let $u = \frac{1}{x}$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$

2. $\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}$ and $v = x$

3. $\int \frac{1}{x}\,\mathrm{d}x = 1 + \int \frac{1}{x}\,\mathrm{d}x$

4. $\cancel{\int \frac{1}{x}\,\mathrm{d}x} = 1 + \cancel{\int \frac{1}{x}\,\mathrm{d}x}$. Therefore, $0 = 1$. QED.

(A) The proof is incorrect, and the first error is on line 1.

(B) The proof is incorrect, and the first error is on line 2.

(C) The proof is incorrect, and the first error is on line 3.

(D) The proof is incorrect, and the first error is on line 4.

(E) The proof is fully correct.

Q10. Find the full range of values of the real number k such that

$$\ln(x)^2 + \ln\left(\frac{1}{x^6}\right) + k = 0$$

has exactly 2 real solutions.

(A) $k > 9$

(B) $k < 9$

(C) $k > 0$

(D) $k < 0$

(E) $0 < k < 9$

(F) $k < 0, k > 9$

Q11. A *repunit* is an integer consisting of only ones. Examples: 1111 or 1 or 11111111.
Complete the sentence: A repunit with n digits ($n > 0$) is divisible by 7 **if and only if...**

(A) n is a multiple of 3

(B) n is a multiple of 6

(C) n is a multiple of 7

(D) n is a multiple of 12

(E) n is of the form $4k + 6$ where k is a non-negative integer

(F) n is of the form $2k + 4$ where k is a non-negative integer

Q12. $f(x) = 5x^2 - x^3 - 7x + 3$, and $0 \leq x \leq 4$. Find the maximum value of $f(x)$ in this range.

(A) $\frac{32}{27}$

(B) 3

(C) 0

(D) 12

(E) $\frac{46}{27}$

(F) $\frac{46}{3}$

(G) -4

Q13. A word is *good* **if and only if** it consists of no letters other than A,B,C,D.
Examples: AABDC or CCCB or AB or ABDBCDA. How many 5-letter *good* words have at least one A and at least one C? (order matters, so ABCDA and ADCBA are distinct)

(A) 0

(B) 32

(C) 160

(D) 256

(E) 570

(F) 813

(G) 1024

(H) 1280

Q14. Which of these numbers is the smallest?

(A) $\ln(9)$

(B) $\pi^{\log_3(2)}$

(C) $\frac{1337}{668}$

(D) $-2\cos(3.14)$

(E) $\sqrt[3]{5}$

(F) $3^{\sin(\frac{25\pi}{6})}$

Q15. Which of these statements is true for two positive integers, p and q, where p is prime?

1. The highest common factor of p and q is 1 **if** q is also prime.

2. pq has exactly four factors

(A) Neither

(B) 1 only

(C) 2 only

(D) Both 1 and 2

Q16. Of the following 7 statements about numbered lamps in a factory, exactly one of them is true. Which one is the true statement? [n is a positive integer]

(A) If n is even, lamp n is switched on.

(B) If n is odd, lamp n is switched off.

(C) If n is even, lamp n is switched off.

(D) If n is odd, lamp n is switched on.

(E) If lamp n is switched on, then n is even.

(F) If lamp n is switched off, then n is odd.

(G) If lamp n is switched off, then n is even.

Q17. For how many of these functions is $f'(x)$ strictly increasing for all real x?

1. $\ln(x)$
2. $\sin(x)$
3. $-\ln(x)$
4. x^2
5. x^3

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

Q18. All 720 permutations of the word "NUMBER" are generated, and arranged in alphabetical order. In what position is the word "NUMBER"

(A) 383rd

(B) 385th

(C) 468th

(D) 469th

(E) 487th

(F) 490th

(G) 618th

(H) 622nd

Q19. A *squarefree* integer is a positive integer which isn't divisible by the square of a prime. Which of these statements about squarefree integers is correct?

1. A squarefree integer with n prime factors has 2^n factors.

2. The product of two squarefree integers is always squarefree

3. A squarefree integer cannot be a power of 36.

(A) None of them

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1,2 and 3

Q20. Solve these simultaneous equations to find the real numbers x, y and z. Hence find $x+y^2+z^3$.

$x^2 + 6y + 10 = 0$
$3y^2 + 6z + 5 = -1$
$40x - 5z^2 = 70$

(A) 4

(B) 6

(C) 10

(D) 12

(E) 14

(F) 16

(G) No solution exists