TMUA Mock Paper 1

20 Questions in the style of a TMUA Paper 1

75 Minutes

No calculator allowed

Enjoy!

(By yotta)

Q1. Find the sum of the x-coordinates of the six points of intersection of

$$y = \pi x \left(x - 1\right) \left(x - 2\right) \left(x - 3\right) \left(x - 4\right) \left(x - 5\right)$$

and

$$y = \frac{5}{17}x + \frac{\pi}{3}$$

(A) -15

(B) $-\frac{44\pi}{51} + 1$

(C) $-\frac{7}{15}$

(D) 0

(E) $\frac{7}{15}$

(F) $\frac{44\pi}{51} - 1$

(G) 12

(H) 15

Q2. A cubic function $f(x)$ is such that $f(2) = 4$, $f(3) = 9$, $f(-1) = 1$, and the coefficient of x^3 is 2. Find $f(4)$.

(A) -16

(B) 6

(C) 16

(D) 32

(E) 36

(F) 46

Q3. Let $f_0(x) = x$, and $f_{n+1}(x) = |f_n(x) - k|$ for non-negative integers n, and real number k. Let α and β respectively equal the least and greatest values of x for which $f_n(x) = 0$. Find the value of:

$$\int_\alpha^\beta f_n(x)\,\mathrm{d}x$$

for $n > 0$, in terms of n and k.

(A) nk^2

(B) $nk^2 - k^2$

(C) kn^2

(D) nk

(E) $k(n-1)^2$

(F) $nk^2 - 1$

Q4. Non-zero integers a, b and c satisfy

$$abc + bc + ab + ac + a + b + c = 104$$

What is $a^2 + b^2 + c^2 + 2(a + b + c) + 1$?

(A) 35

(B) 54

(C) 56

(D) 81

(E) 104

(F) 105

Q5. Let $u_n = 2u_{n-1} + 7u_{n-2}$, where $u_1 = 4$ and $u_2 = 12$. What does the value of $\frac{u_k}{u_{k-1}}$ tend towards as k tends towards infinity?

(A) 2

(B) $\sqrt{3}$

(C) $\sqrt{5} + 1$

(D) 7

(E) 9

(F) $1 + 2\sqrt{2}$

(G) $-1 + \sqrt{3}$

Q6. $f(x)$ is a polynomial function defined for all real x. Given that $f(x^2 - 12x + 45)$ has two roots at $x = -3$ and $x = 15$, and has a minimum value of -20, which row correctly describes $f(9x^2 - 30x + 34)$?

	Roots	Min Value
(A)	$x = -1$ and $x = 5$	-20
(B)	$x = -\frac{4}{3}$ and $x = \frac{14}{3}$	-20
(C)	$x = -\frac{2}{3}$ and $x = \frac{16}{3}$	-20
(D)	$x = -1$ and $x = 42$	-20
(E)	$x = -12$ and $x = 5$	-60
(F)	$x = -1$ and $x = 42$	-180

Q7. How many real solutions are there to

$$2(27^x) - 3^{2x+1} - 4(3^{x+1}) + 5 = 0$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

Q8. The graph of $y = \tan(\cos(\sin(x)))$ has a period of P and a maximum value of M. Which row is correct?

(A)	$P = 1$	$M < 1$
(B)	$P = 1$	$M > 1$
(C)	$P = \frac{\pi}{2}$	$M < 1$
(D)	$P = \frac{\pi}{2}$	$M > 1$
(E)	$P = \pi$	$M < 1$
(F)	$P = \pi$	$M > 1$
(G)	$P = 2\pi$	$M < 1$
(H)	$P = 2\pi$	$M > 1$

Q9. How many real solutions are there to

$$\ln(\sin(x)) = \ln\left(1 - \frac{4x}{7\pi}\right)$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

(H) infinitely many

Q10. A fair coin is flipped repeatedly until 4 consecutive heads are obtained. Find the expected number of coin flips.

(A) 4

(B) 14

(C) 16

(D) 30

(E) $\frac{196}{5}$

(F) $\frac{288}{7}$

(G) 62

(H) 64

Q11. Worker n, where n is an integer, can do a task by themself in 2^n days. Let $f(k)$ represent the time taken when workers 0 to k inclusive are all working on the task simultaneously (assuming their overall speed adds up). What is the value of $f^{-1}(\frac{255}{512})$?

(A) 8

(B) 9

(C) 16

(D) 17

(E) 64

(F) 65

(G) Doesn't exist

Q12. Which of these values is the smallest?

(A) $\sin(1)$

(B) $\cos(\frac{1}{2})$

(C) 0.88

(D) $\frac{1}{2}\tan(\frac{\pi}{3})$

(E) $\frac{2}{\sqrt{5}}$

Q13. Find the sum of the reciprocals of all of the factors of 1600.

(A) $\frac{1}{3937}$

(B) $\frac{1600}{3937}$

(C) $\frac{3937}{1600}$

(D) $\frac{378}{1600}$

(E) $\frac{1600}{378}$

(F) $\frac{1}{378}$

Q14. The digital root of a number is where you find the sum of the digits of a number, then find the sum of the answer, and repeat until you get a 1-digit number. For example, to find the digital root of 9678996 you do $9+6+7+8+9+9+6=54$, $5+4=9$, so its digital root is 9. What's the digital root of 7^{3935}?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

(G) 7

(H) 8

Q15. The function $f(x)$ has the property that $f(x) = f(6-x)$ for all real x. Given that:

$$\left(\int_2^3 f(x)\,\mathrm{d}x\right)^2 + \left(\int_4^6 f(x)\,\mathrm{d}x\right)^2 + \left(\int_2^4 f(x)\,\mathrm{d}x\right)\left(\int_0^2 f(x)\,\mathrm{d}x\right) - \int_6^3 3f(x)\,\mathrm{d}x = -2$$

Find the sum of the possible values of $\int_0^3 f(x)\,\mathrm{d}x$.

(A) -10

(B) -5

(C) -3

(D) 0

(E) 3

(F) 5

(G) 10

(H) $\frac{27}{2}$

Q16. A point A is chosen on the curve with equation:

$$(x-2)^2 + (y-3)^2 = 4$$

and another point B is chosen on the curve with equation:

$$x^2 + y^2 + 8x + 10y = r$$

Find the length of the interval within the range $0 < r < 125$ for which the shortest possible distance of AB is less than 1.

(A) 3

(B) 5

(C) 6

(D) 49

(E) 76

(F) 117

(G) 119

(H) 120

Q17. $f(x) = 1!x^{2!x^{3!x^{4!x^{5!x^{6!x}}}}}$. Which of these is the closest value of $f(\frac{1}{2})$?

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) 1

(F) 2

(G) 1000000

Q18. 45 people are standing in a line. How many ways are there to choose 17 of them such that no two chosen people are next to each other, and order doesn't matter?

(A) $\frac{45!}{28!17!}$

(B) $\frac{43!}{26!17!}$

(C) $\frac{28!}{17!11!}$

(D) $\frac{29!}{17!12!}$

(E) $\frac{17!12!}{8!}$

(F) $\frac{2^{17}45!}{12!}$

Q19. Consider $f(x) = \ln(\sqrt{x^2 - 2x + 1})$. Which of the following statements is true about $f(x)$?

(A) It is defined for all real x.

(B) Defined for all real $x < 0$ but not all real $x > 0$.

(C) Defined for all real $x > 0$ but not all real $x < 0$.

(D) Undefined for $x = e$.

(E) $8 < f^{-1}(2) < 9$.

(F) $f(x) = 100$ has no solutions.

Q20. The function $F(x)$, where x is a non-negative real number, is the result of subtracting the integer part of x from x. For example, $F(3) = 0$, $F(5.43) = 0.43$, $F(\pi) = 0.14159265...$, Find an expression for:

$$\int_0^{\sqrt{k}} F(x^2)\,\mathrm{d}x$$

where k is a positive integer.

(A) $\frac{1}{3}k^{\frac{3}{2}} - F(\frac{1}{3}k^{\frac{3}{2}})$

(B) $\frac{1}{3}k^{\frac{3}{2}} - \sum_{n=0}^{k} n(\sqrt{n+1} - \sqrt{n})$

(C) $\frac{1}{3}k^{\frac{3}{2}} + \sum_{n=0}^{k-1} n(\sqrt{n+1} - \sqrt{n})$

(D) $\frac{1}{3}k^{\frac{3}{2}} - \sum_{n=0}^{k-1} n(\sqrt{n+1} + \sqrt{n})$

(E) $\frac{1}{3}k^{\frac{3}{2}} + (\sum_{n=0}^{k-1} \sqrt{n}) + (k-1)\sqrt{k}$

(F) $\frac{1}{3}k^{\frac{3}{2}} + (\sum_{n=0}^{k-1} \sqrt{n}) + (1-k)\sqrt{k}$